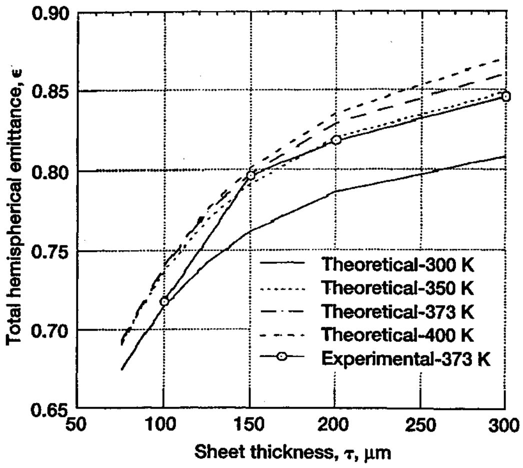

Fig. 2 Comparison of theoretical and experimental total hemispherical emittance for Dow Corning 705 oil.

ues for all sheet thicknesses. As shown in Fig. 2, the emittance increases with an increase in temperature and an increase in sheet thickness. As the temperature increases past 400 K, the blackbody hemispherical spectral power is a maximum at wavelengths shorter than 9–10 μm where spectral emittance is a maximum. Thus, the overall hemispherical emittance will begin to decrease as the temperature increases beyond 400 K. As the sheet thickness is increased, the emittance will level off and reach a constant value.

Conclusion

The emittance of a thin liquid sheet flow of Dow Corning 705 oil was determined by two methods. From the analysis and results, several points can be made. It is evident that the Dow Corning 705 liquid sheet has a strong potential of functioning well as a radiator. The experimental emittance and the calculated emittance show high values averaging from 0.74 to 0.85. Because reflectance was neglected in Eq. (1), the calculated emittance has an error of approximately 5%. Experimentally, it was difficult to determine if the temperature fluctuations were insignificant, but efforts were made to eliminate any fluctuations. Thus far, research on the liquid sheet indicates it is an excellent candidate for a low mass space radiator.

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Comparison of Numerical Quadrature Schemes Applied in the Method of Discrete Transfer

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Introduction

RADIATION heat transfer modeling is complicated by the larger number of independent variables: 1) three space coordinates describing the location, 2) two angles describing the direction, and 3) sometimes several wavelength intervals. In some applications, where the changes are fast compared to the propagation time within the geometry of the thermal radiation, the time is an independent variable. The high number of independent variables makes the solution strategy very important to obtain fast and accurate solutions of the radiative heat transfer problem.

In the method of discrete transfer, the irradiation to wall zones is calculated by a numerical quadrature that is close to the so-called midpoint method.[1,2] Although the method is very successful, other numerical quadrature schemes should be considered for possible use to reduce the calculation costs for a given numerical accuracy, or for the same computational costs, to obtain a more accurate result; this is the theme of this Note.

The geometry chosen is an infinitely long, circular cylinder with cold, black boundaries filled with an absorbing, emitting gas with constant temperature and radiative properties (see Fig. 1). The irradiation to the cylindrical wall is calculated. The analyses are valid for gray gases as well as for the monochromatic cases.

Several other numerical methods are applicable within the field of thermal radiation and some excellent references are Siegel and Howell,[3] Viskanta,[4] Viskanta and Menguc,[5] and Viskanta and Ramadhyani.[6]

Theory

The method of discrete transfer is a ray-tracing technique for solving the radiative transfer problem primarily in absorbing and emitting gas[1] where the irradiation H is found by numerical integration of

$$H = \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi/2} I \cdot \cos\theta \cdot \sin\theta \cdot d\theta \, d\phi \qquad (1)$$

$$H = \int_{\phi=0}^{2\pi} \int_{\mu=0}^{1} I \cdot \mu \cdot d\mu \, d\phi \qquad (2)$$

$$H = \int_{\phi=0}^{2\pi} \int_{\kappa=0}^{1/2} I \cdot d\kappa \, d\phi \qquad (3)$$

where θ is the angle from the normal of the surface to the considered direction, ϕ is the azimuthal angle (see Fig. 1), $\mu = \cos\theta$, and $\kappa = \frac{1}{2}\mu^2$. The incident intensity is denoted I. Further details of the method of discrete transfer are given in Refs. 1 and 2.

The incident intensity in a given direction is

$$I = I_b \cdot [1 - \exp(-a \cdot l)] \qquad (4)$$

Received Aug. 31, 1995; revision received Jan. 25, 1996; accepted for publication March 11, 1996.

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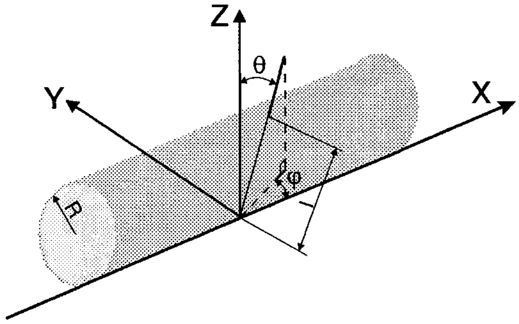

Fig. 1 Schematic of infinite cylinder.

where a is the absorption coefficient, I_b is the blackbody intensity, and l is the length of the ray evaluated from

$$l = \frac{2 \cdot R \cdot \cos\theta}{\cos^2\theta + \sin^2\theta \cdot \sin^2\phi} \tag{5}$$

where R is the radius of the circular cylinder of gas (see Fig. 1).

To reduce the number of variables, a dimensionless irradiation is introduced:

$$H^* = H/(\pi \cdot I_b) \tag{6}$$

Applying Eqs. (4) and (6) to Eqs. (1–3), the dimensionless irradiation is

$$H^* = \frac{1}{\pi} \cdot \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi/2} [1 - \exp(-a \cdot l)] \cdot \cos\theta \cdot \sin\theta \cdot d\theta \, d\phi \tag{7}$$

$$H^* = \frac{1}{\pi} \cdot \int_{\phi=0}^{2\pi} \int_{\mu=0}^{1} [1 - \exp(-a \cdot l)] \cdot \mu \cdot d\mu \, d\phi \tag{8}$$

$$H^* = \frac{1}{\pi} \cdot \int_{\phi=0}^{2\pi} \int_{\kappa=0}^{1/2} [1 - \exp(-a \cdot l)] \cdot d\kappa \, d\phi \tag{9}$$

Substituting Eq. (5) into Eqs. (7–9) shows that the dimensionless irradiation depends on one variable only, namely $R \cdot a$, which is the dimensionless radius of the gas cylinder and a measure of the optical thickness.

Integration Quadrature Techniques

Shah Quadrature Scheme

The integral in Eq. (7) is approximated by a sum, as follows, since the θ and ϕ angles are discretized into equal subangles[1,2,7]:

$$H^* \approx \frac{1}{\pi} \cdot \sum_j [1 - \exp(-a \cdot l)] \cdot \cos\theta_j \cdot \sin\theta_j \cdot \sin\Delta\theta \cdot \Delta\phi \tag{10}$$

The angles θ_j and ϕ_j are equidistant. The results when applying the Shah quadrature are denoted Shah.

Gauss–Legendre Quadrature Scheme

A Fortran subroutine for calculating the ordinates and the weights in the Gauss–Legendre scheme is described in Press et al.[8] and given in Vetterling et al.,[9] and is used in the present work either directly, when single precision (real*4) is required, or in a modified version rewritten to double precision when that is required.

From the one-dimensional Gauss–Legendre, the two-dimensional quadrature scheme for integration over a quadrate is calculated: the ordinates in the two directions are found directly and the weights are found by multiplication of the appropriate weights of the one-dimensional cases.[10]

The two-dimensional Gauss–Legendre is applied in this case without any coordinate transformation (only scaling and moving the coordinate system).

The Gauss–Legendre quadrature is applied to Eqs. (7–9), and the results are called Gauss-θ, Gauss-μ, and Gauss-κ, respectively.

Discrete Ordinates Quadrature

The ordinates and weights for low-order approximations S-2, S-4, S-6, and S-8 are copied from Fiveland[11] (see also Ref. 12). It is emphasized that the method of discrete ordinates is not applied; only the quadrature schemes from the method are applied in the method of discrete transfer. For the S-2, S-4, S-6, and S-8 sets, the number of rays tracked is 4, 12, 24, and 40.

Midpoint Quadrature

The midpoint quadrature scheme has been applied to Eq. (7) as follows:

$$H^* \approx \frac{1}{\pi} \cdot \sum_j [1 - \exp(-a \cdot l)] \cdot \cos\theta_j \cdot \sin\theta_j \cdot \Delta\theta \cdot \Delta\phi \tag{11}$$

Discussion

The calculations are performed in real precision (real*4), unless otherwise indicated. In the Shah and Gauss–Legendre quadratures, the number of rays in the ϕ direction is $N_\phi = 4N_\theta$, and thus the total number of rays is $N_{tot} = 4N_\theta^2$.

To investigate the effect of the number of digits in the calculations, the dimensionless irradiation is found using real*4 and real*8 for both the Shah quadrature and the Gauss-θ quadrature. The single- and double-precision operators give the same results when the total number of rays is less than approximately 2500; when the number of rays is much higher than 2500, there may be a difference because of error cumulation. In the present case, it is acceptable to use single precision (real*4) numbers, unless a very large number of rays is required.

The Shah quadrature in double precision is applied for calculating a very accurate value of the dimensionless irradiation for $Ra = 1.0$ using 340,000 rays and H^* is found to be 0.8142891. Furthermore, Monte Carlo calculations were performed for comparison. To obtain information about the statistical accuracy of the results, 10 calculations were made in each case, each applying 10^8 bundles. The average value for the dimensionless irradiation is 0.8142291, and the standard deviation of the average is 4.2×10^{-5}. The dimensionless irradiation calculated by the method of discrete transfer is in agreement with the result of the Monte Carlo method considering the calculated standard deviations of the Monte Carlo results. The very accurate value for H^* is applied for calculating the errors of the results obtained by the various methods in the following.

The optical thin limit is obtained for $2Ra \rightarrow 0$ and, because the geometrical mean beam length is $2R$, the dimensionless irradiation is $2Ra$, which is indicated in the results of the integrations since the dimensionless irradiation is 0.019739 for $2Ra = 0.02$. In the optical thick case, the intensity is independent of the direction and equals the blackbody intensity. Equations (7–9) yield the dimensionless irradiation of 1. For $Ra = 10.0$, the dimensionless irradiation is integrated to be 0.99811 and for $Ra = 100$, the dimensionless irradiation is 0.99989.

In Table 1 the absolute errors of the dimensionless irradiation for $Ra = 1.0$ are shown for all of the investigated quadrature schemes except the scheme from the method of discrete ordinates, and the cases with the lowest errors are indicated by the footnote symbol a. The Gauss-μ is a good scheme, although it does not always give the smallest errors.

The errors when using the quadratures from the method of discrete ordinates in the method of discrete transfer are shown

Table 1 Dimensionless irradiation error calculated by various quadrature schemes for Ra = 1.0 and various numbers of rays

	Error of H^*				
Ra = 1.0	Shah	Midpoint	Gauss-θ	Gauss-μ	Gauss-κ
N_{tot}					
16	−0.0015341	0.0884547	−0.0446258	−0.0069799[a]	0.0138574
64	−0.0041260	0.0170778	0.0000320[a]	0.0001676	0.0035644
196	−0.0018838	0.0049743	−0.0000119	−0.0000057[a]	0.0008997
2,500	−0.0001963	0.0003375	0.0000028	0.0000005[a]	0.0000476
160,000	−0.0000362	−0.0000280	−0.0000500	0.0000029	0.0000006[a]

[a]Indicates the lowest error for a given number of rays.

Table 2 Absolute error of the dimensionless irradiation[a]

Quadrature	Error of H^*
S-2	0.1361203
S-4	−0.0068372
S-6	−0.0066329
S-8	0.0026947

[a]Calculated using the quadrature scheme from the method of discrete ordinates for Ra = 1.0.

in Table 2. The error of the S-8 approximation (40 rays) is 0.0027, which is comparable to the Shah scheme with approximately the same number of rays.

The drawbacks and advantages of the methods may to some extent be explained by the fact that some of the methods integrate the first moment exactly and others do not. The mth-order moment is defined as

$$\int_{2\pi} \cos^m\theta \cdot d\omega \qquad (12)$$

Gauss-θ and the midpoint quadratures do not integrate the first moment exactly. Shah, Gauss-μ, and Gauss-κ all integrate the first moment exactly. Gauss-μ and discrete ordinate quadrature have the further advantage of integrating the zeroth moment exactly, which the other methods do not. For N_{tot} = 16, the disadvantage with the first-order moment is reflected in Table 1 where the Gauss-θ and midpoint quadratures give the largest errors. The quadrature scheme from the method of discrete order does not integrate the first moment exactly in the S-2 approximation (S-4, S-6, and S-8 integrate exactly both the zeroth and first moments) and that disadvantage is also shown in Table 2, where the error is large for the S-2 approximation.

Conclusions

Several numerical quadrature schemes are compared for an infinitely long circular cylinder of absorbing, emitting, constant property gas. The quadrature schemes of Shah, Gauss–Legendre, midpoint, and method of discrete ordinates are applied for calculating the dimensionless irradiation to the boundary of the circular cylinder. The Gauss–Legendre quadrature scheme is applied in three forms of the integral equation for the dimensionless irradiation.

The quadrature scheme Gauss-μ often gives higher accuracy, but is slightly more complex to apply than the quadrature scheme originally suggested by Shah. The Gauss-θ should not be applied for a low number of rays because the quadrature scheme does not satisfy the first-order moment.

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